Multiple Bit Differential Detection of Offset Quadrature Phase-Shift-Keying

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Abstract—**Analogous to multiple symbol differential detection of quadrature phase-shift-keying (QPSK), a multiple bit differential detection scheme is described for offset QPSK that also exhibits continuous improvement in performance with increasing observation interval. Being derived from maximum-likelihood (ML) considerations, the proposed scheme is purported to be the most power efficient scheme for such a modulation and detection method. Extension of the results to shaped offset QPSK is also possible.**

I. Introduction

More than a decade ago, multiple symbol differential detection of M-ary phase-shift-keying (M-PSK) [1] was introduced by the author as a means of improving system performance relative to the traditional (two-symbol observation) differential detection scheme. The technique made use of maximum-likelihood sequence estimation (MLSE) of the transmitted phases rather than symbol-by-symbol detection and, depending on the number of symbols observed, its performance was shown to span between that of conventional differential detection and ideal coherent detection of differentially encoded M-PSK. One special case of high interest corresponds to $M = 4$, i.e., quadrature phase-shift-keying (QPSK) and numerical results were reported in [1] for this case to allow comparison with conventional differential detection of QPSK (DQPSK). By comparison, the literature is quite sparse [2,3] regarding differential detection of offset QPSK (OQPSK) despite the fact that OQPSK has a much higher spectral containment than non-offset QPSK when transmitted over bandlimited nonlinear channels. As a compromise between these two spectral efficiencies, $\pi/4$-DQPSK [4] was proposed whose detection can be performed by a straightforward modification of the techniques used for conventional DQPSK and also for multiple symbol detection of $\pi/4$-DQPSK [5]. While $\pi/4$-DQPSK offered a modest improvement in spectral containment over QPSK (the maximum instantaneous phase transitions are reduced from 180° for the latter to 135° for the former) at little or no sacrifice in power efficiency, it was still a far cry from the spectral efficiency achieved by OQPSK. Understanding that, because of the inherent crosstalk between quadrature channels introduced by the lack of absolute phase knowledge associated with differential detection, one would expect to pay a power penalty when differentially detecting OQPSK (DOQPSK), the author set out to find the "best" one could do in this regard. Specifically, by applying the same MLSE principle used to achieve the performance enhancement of DQPSK attained in [1], we derive an analogous multiple observation interval differential detection technique for OQPSK and examine its behavior in the limit of large observation time.

We start by identifying an equivalent precoded continuous phase modulation (CPM) structure first for OQPSK and then next for differentially encoded OQPSK. Following this, we recall the results of the author for ML block detection of noncoherent CPM reported in [6] and then apply the technique used there to derive the decision metric and associated receiver structure for the precoded version that equivalently represents differentially encoded OQPSK. Finally, we evaluate (in terms of upper bounds) the average bit error probability performance of this multiple bit DOQPSK scheme for cases of practical interest and compare it with the analogous results for non-offset QPSK.

II. Precoded CPM Equivalent of OQPSK and Differentially Encoded OQPSK

In this section, we describe a representation of conventional OQPSK (rectangular pulse shaping implied) in the form of a precoded CPM modulation. Specifically, OQPSK has the form

$$s(t)=\sqrt{\frac{2E_b}{T_b}}\cos\left(2\pi f_c t+\phi(t,\boldsymbol{\alpha})+\phi_0\right),\quad nT_b\le t\le(n+1)T_b \qquad (2.1)$$

where E_b and T_b respectively denote the energy and duration of a bit ($P=E_b/T_b$ is the signal power), and f_c is the carrier frequency. In addition, $\phi(t,\boldsymbol{\alpha})$ is the phase modulation process that is expressible in the form

$$\phi(t,\boldsymbol{\alpha})=\pi\sum_{i\le n}\alpha_i q\left(t-iT_b\right) \qquad (2.2)$$

where $\boldsymbol{\alpha}=\left(\ldots,\alpha_{-2},\alpha_{-1},\alpha_0,\alpha_1,\alpha_2,\ldots\right)$ is a precoded version of the true data sequence and $q(t)$ is the normalized phase-smoothing response that defines how the underlying phase, $\pi\alpha_i$, evolves with time during the associated bit interval. Without loss of generality, the arbitrary phase constant, ϕ_0, can be set to zero. For OQPSK, the phase pulse $q(t)$ is a step function, i.e., $q(t)=(1/2)u(t)$ [equivalently, the frequency pulse $g(t)=dq(t)/dt$ is the impulse function

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$g(t) = (1/2)\delta(t)]$ and the ith element of the CPM data sequence, α_i, can be shown to be related to the true input data bit sequence $\mathbf{a} = (\ldots, a_{-2}, a_{-1}, a_0, a_1, a_2, \ldots)$ by [7, Chap. 3, pp. 177–178]

$$\alpha_i = (-1)^{i+1} \frac{a_{i-1}(a_i - a_{i-2})}{2} \tag{2.3}$$

Since the a_i's take on ± 1 values, then the α_i's come from a ternary (–1,0,+1) alphabet. However, in any given bit (half-symbol) interval, the α_i's can only assume one of two equiprobable values, namely, 0 and +1 or 0 and –1, with the further restriction that a +1 cannot be followed by a –1, or vice versa. Thus, in reality, the modulation scheme is a binary CPM but one whose data alphabet can vary (between two choices) from bit interval to bit interval. Since the duration of the frequency pulse does not exceed the baud (bit) interval, then the CPM representation of OQPSK is full response and can be implemented with the cascade of a precoder satisfying (2.3) and a conventional CPM modulator.

In order to find a precoded CPM representation for differentially encoded OQPSK, we recall that if $\{b_n\}$ is a binary (± 1) independent and identically distributed (i.i.d.) sequence, then $\{a_n\}$ with elements $a_n = b_n a_{n-1}$ is the differentially encoded version of $\{b_n\}$ and is also i.i.d. Alternatively, since $b_n = a_n a_{n-1}$, then the precoder of (2.3) can be rewritten in terms of the b_n's as

$$\alpha_i = (-1)^{i+1} \frac{b_i - b_{i-1}}{2} \tag{2.4}$$

Thus, a precoded CPM representation of differentially encoded OQPSK would employ the precoder of (2.4) instead of that in (2.3).

Since the noncoherent demodulator of the CPM modulation will result in decisions $\{\hat{\alpha}_n\}$ on the symbols $\{\alpha_n\}$, then in order to convert these decisions into ones on the true input binary data sequence ($\{b_n\}$ for differentially encoded OQPSK), one would have to follow the CPM demodulator with a decoder that reverses the precoding operation in (2.4). Rather than do that, one can include an additional differential encoding operation at the transmitter in such a way that the decisions $\{\hat{\alpha}_n\}$ on the symbols $\{\alpha_n\}$ will now directly reflect decisions on the true binary data input. To see how this can be accomplished, we define

$$c_n = 1 - 2|\alpha_n| = 1 - |b_n - b_{n-1}| \tag{2.5}$$

Thus, $c_n = -1$ if b_{n-1} makes a transition and $c_n = 1$ if b_{n-1} does not make a transition. Since the relation between c_n and b_n is again that of conventional differential encoding, we see that decisions $\{\hat{c}_n\}$ derived from the CPM demodulator decisions $\{\hat{\alpha}_n\}$ in accordance with (2.5) will represent decisions on an input data sequence $\{c_n\}$ whose differentially encoded version is $\{b_n\}$. The inclusion of this additional differential encoder at the input of the OQPSK modulator results in a transmitter that implements OQPSK with a *double* differential encoder of its input binary data sequence (Fig. 1). However, double differentially encoding the binary input sequence prior to demultiplexing into inphase (I) and quadrature (Q) sequences is exactly equivalent to first demultiplexing the input sequence and then differentially encoding the binary I and Q symbols [each of duration $2T_b$ and offset with respect to one another. Thus, the CPM receiver illustrated in Fig. 1 is an appropriate demodulator of what is conventionally known as differentially encoded OQPSK.

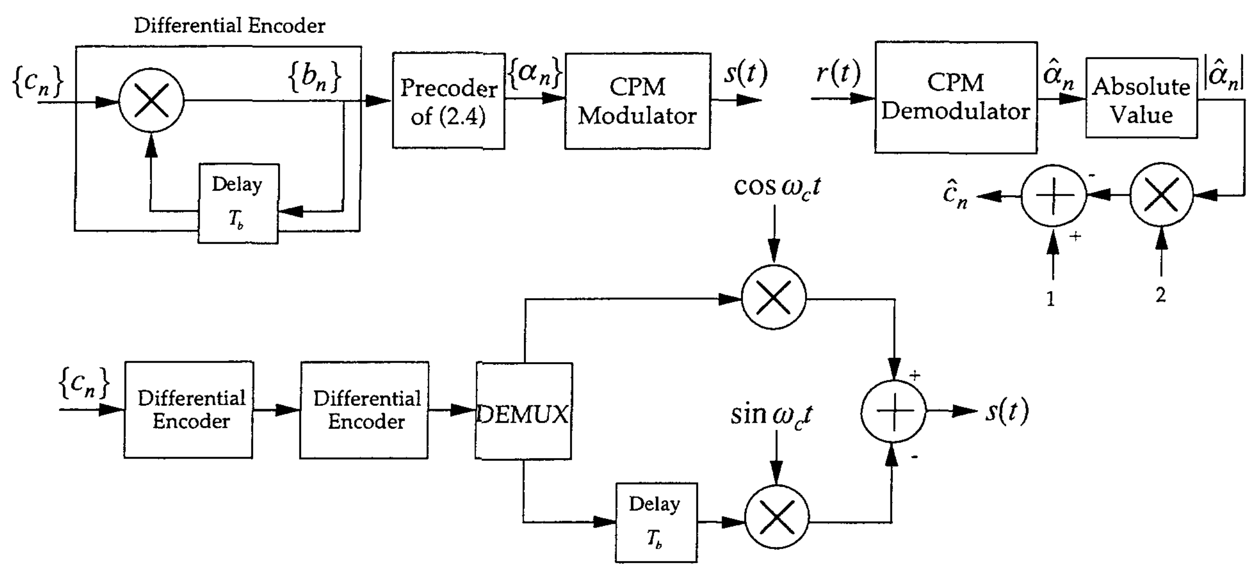

Fig. 1. Equivalent CPM and OQPSK with double differentially encoded binary input stream transmitters and also noncoherent CPM demodulator

III. Maximum-Likelihood Sequence Detection of Noncoherent Precoded CPM

Expressing (2.1) in complex baseband form, i.e., $s(t) = \mathrm{Re}\left\{\tilde{S}(t)e^{j\omega_c t}\right\}$ where $\tilde{S}(t) = \sqrt{2E_b / T_b}\, e^{j\phi(t,\boldsymbol{\alpha})}$, then transmitting $\tilde{S}(t)$ over an additive white Gaussian noise (AWGN) channel results in a received complex baseband signal $\tilde{R}(t)$ of the form

$$\tilde{R}(t) = \tilde{S}(t)e^{j\theta(t)} + n(t) \tag{3.1}$$

where $n(t)$ is a zero mean complex Gaussian noise process with two-sided power spectral density $2N_0$ W/Hz and $\theta(t)$ is an arbitrary phase introduced by the channel which is assumed to be constant (independent of time) over some specified interval of time, i.e., $\theta(t) = \theta$ but is otherwise unknown. Furthermore, in the absence of any side information, θ is assumed to be uniformly distributed in the interval $(-\pi, \pi)$. Following the approach taken in [6], for an N-bit observation, the MLSE decision rule for jointly detecting the data sequence $\boldsymbol{\alpha} = \alpha_{n-N+1}, \alpha_{n-N+2}, \ldots, \alpha_{n-1}, \alpha_n$ is given by

$$\text{Choose } \boldsymbol{\alpha} = \hat{\boldsymbol{\alpha}} \text{ corresponding to } \left|\beta(\hat{\boldsymbol{\alpha}})\right| = \max_{\boldsymbol{\alpha}} \left|\beta(\boldsymbol{\alpha})\right| \tag{3.2}$$

where

$$\beta(\boldsymbol{\alpha}) = \sum_{l=0}^{N-1} \Gamma_{n-l} C_{n-l} \tag{3.3}$$

with

$$\Gamma_n = \int_{nT_b}^{(n+1)T_b} \tilde{R}(t)\,dt \tag{3.4}$$

representing the observation corresponding to the nth bit interval, i.e., the complex output of an integrate-and-dump (I&D) filter, and the coefficients $\{C_n\}$ defined recursively by

$$\begin{aligned} C_{n-l} &= e^{-j(\pi/2)\alpha_{n-l}} C_{n-l-1}, \; l = 0, 1, \ldots, N-2, \\ C_{n-N+1} &= e^{-j(\pi/2)\alpha_{n-N+1}} \end{aligned} \tag{3.5}$$

Since the decision rule in (3.2) only involves the magnitude of $\beta(\boldsymbol{\alpha})$, then noting that the factor $\exp[-j(\pi/2)\alpha_{n-N+1}]$ is common to each term of the sum in (3.3), we obtain

$$\left|\beta(\boldsymbol{\alpha})\right| = \left| \sum_{l=0}^{N-1} \Gamma_{n-l}\, j^{-\sum_{k=l}^{N-2} \alpha_{n-k}} \right| \tag{3.6}$$

Thus we observe from (3.6) that an observation of N bits actually results in a decision on only the $N-1$ most recent bits, $\alpha_{n-N+2}, \ldots, \alpha_{n-1}, \alpha_n$ as was the case for the multiple symbol differential detection scheme described in [1]. Finally, to arrive at decisions on the true input data stream, $\{c_n\}$, the decision rule of (3.2) is modified in accordance with (2.5) to become

$$\text{Choose } \mathbf{c} = \hat{\mathbf{c}} = 1 - 2|\hat{\alpha}| \text{ corresponding to } \left|\beta(\hat{\alpha})\right| = \max_{\alpha} \left|\beta(\alpha)\right| \tag{3.7}$$

IV. Evaluation of an Upper Bound on Average Bit Error Probability

To evaluate the performance of the receiver in Fig. 2, we make use of the technique in [1] to obtain an upper (union) bound on the average bit error probability (BEP) in the form of a sum of the pairwise error probabilities (PEP) associated with each N-bit error block. For our case, the PEPs can be evaluated exactly using the results of Stein [8] as applied to the noncoherent CPM problem in [9].

Let $\mathbf{c} = (c_{n-N+2}, c_{n-N+3}, \ldots, c_{n-1}, c_n)$ denote the sequence of $N-1$ information bits and $\hat{\mathbf{c}} = (\hat{c}_{n-N+2}, \hat{c}_{n-N+3}, \ldots, \hat{c}_{n-1}, \hat{c}_n)$ be the corresponding sequence of detected bits. Then,

$$P_b(E) \le \frac{1}{N-1} \frac{1}{2^{N-1}} \sum_{\mathbf{c} \ne \hat{\mathbf{c}}}\sum w(\mathbf{c}, \hat{\mathbf{c}}) P(\mathbf{c}, \hat{\mathbf{c}}) \Pr\left\{\left|\hat{\beta}\right| > \left|\beta\right| \big| \mathbf{c}\right\} \tag{4.1}$$

where $w(\mathbf{c}, \hat{\mathbf{c}})$ denotes the Hamming distance between $\mathbf{c}$ and $\hat{\mathbf{c}}$, $\Pr\left\{\left|\hat{\beta}\right| > \left|\beta\right| \big| \mathbf{c}\right\}$ denotes the PEP that $\hat{\mathbf{c}}$ is chosen when $\mathbf{c}$ is sent, and $P(\mathbf{c}, \hat{\mathbf{c}}) = 1/N_e(\mathbf{c}, \hat{\mathbf{c}})$ where $N_e(\mathbf{c}, \hat{\mathbf{c}})$ is the number of different error sequence pairs that have to be considered for a particular $(\mathbf{c}, \hat{\mathbf{c}})$.[1] Note that $\sum_{\mathbf{c} \ne \hat{\mathbf{c}}}\sum P(\mathbf{c}, \hat{\mathbf{c}}) = 2^{N-1}\left(2^{N-1} - 1\right)$.

A. Evaluation of the Pairwise Error Probability

We use the approach taken in [6] to compute $\Pr\left\{\left|\hat{\beta}\right| > \left|\beta\right| \big| \mathbf{c}\right\}$, or equivalently, $\Pr\left\{\left|\hat{\beta}\right|^2 > \left|\beta\right|^2 \big| \mathbf{c}\right\}$, which is in turn based on the approach used in [8] to evaluate the performance of noncoherent FSK. Specifically, letting $\eta = |\beta|^2$ and $\hat{\eta} = \left|\hat{\beta}\right|^2$, then

$$\Pr\{\hat{\eta} > \eta | \mathbf{c}\} = \frac{1}{2}\left[1 - Q\left(\sqrt{b}, \sqrt{a}\right) + Q\left(\sqrt{a}, \sqrt{b}\right)\right] \tag{4.2}$$

where $Q(a, b)$ is the first order Marcum Q-function

$$\begin{Bmatrix} b \\ a \end{Bmatrix} = \frac{E_b}{2N_0}\left(N \pm \sqrt{N^2 - |\delta|^2}\right) \tag{4.3}$$

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[1] Note that for the analogous M-DQPSK problem considered in [1], $N_e(\mathbf{c}, \hat{\mathbf{c}}) = 1$ for all $\mathbf{c}$ and $\hat{\mathbf{c}}$ and thus the term $P(\mathbf{c}, \hat{\mathbf{c}})$ was absent in the union bound on BEP.

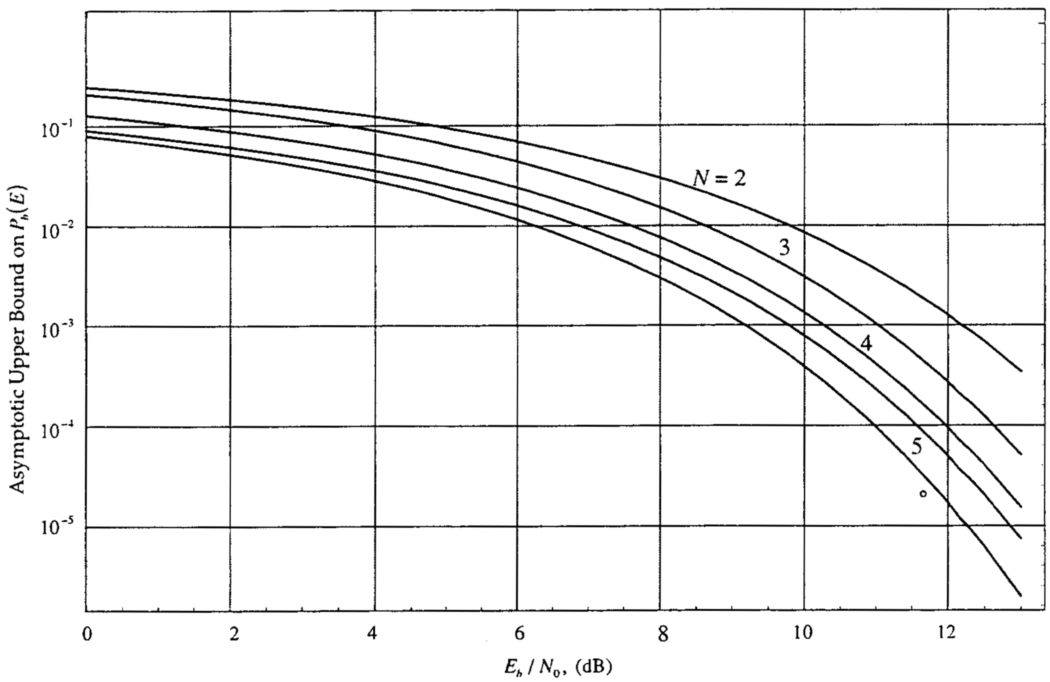

Fig. 2. Asymptotic upper bound on average BEP versus bit energy-to-noise ratio in dB.

with E_b / N_0 denoting the bit signal-to-noise ratio (SNR) and

$$\delta \triangleq \sum_{l=0}^{N-1} j^{\sum_{m=0}^{N-l-2}\left(\alpha_{n-l-m}-\hat{\alpha}_{n-l-m}\right)} = \sum_{l=0}^{N-1} j^{\sum_{m=0}^{N-l-2}\Delta\alpha_{n-l-m}} \tag{4.4}$$

It is understood that the summation in the exponent evaluates to zero if the upper index is negative.

As examples, we give the results for $N = 2$ and $N = 3$.

$$P_b(E)\big|_{N=2} \le \frac{1}{2}\left[1 - Q\left(\sqrt{\frac{E_b}{2N_0}\left(2+\sqrt{2}\right)}, \sqrt{\frac{E_b}{2N_0}\left(2-\sqrt{2}\right)}\right) + Q\left(\sqrt{\frac{E_b}{2N_0}\left(2-\sqrt{2}\right)}, \sqrt{\frac{E_b}{2N_0}\left(2+\sqrt{2}\right)}\right)\right] \tag{4.5}$$

$$P_b(E)\big|_{N=3} \le PEP_1 + PEP_2 \tag{4.6}$$

with

$$PEP_1 = \frac{1}{2}\left[1 - Q\left(\sqrt{\frac{E_b}{N_0}\left(\frac{3}{2}+\sqrt{2}\right)}, \sqrt{\frac{E_b}{N_0}\left(\frac{3}{2}-\sqrt{2}\right)}\right) + Q\left(\sqrt{\frac{E_b}{N_0}\left(\frac{3}{2}-\sqrt{2}\right)}, \sqrt{\frac{E_b}{N_0}\left(\frac{3}{2}+\sqrt{2}\right)}\right)\right] \tag{4.7}$$

$$PEP_2 = \frac{1}{2}\left[1 - Q\left(\sqrt{\frac{5E_b}{2N_0}}, \sqrt{\frac{E_b}{2N_0}}\right) + Q\left(\sqrt{\frac{E_b}{2N_0}}, \sqrt{\frac{5E_b}{2N_0}}\right)\right]$$

Comparing (4.5) with the optimum average BEP performance of DQPSK (which is exactly given by the right hand side of (4.5) with E_b replaced by $E_s = 2E_b$), we note that, for a two-bit observation interval, the performance of the DOQPSK receiver is at most 3 dB worse (based on the upper bound). In fact, it is not difficult to show that the upper bound of (4.5) is in fact equal to the actual average BEP performance of the DOQPSK receiver and thus the penalty relative to DQPSK is *exactly* 3 dB. This should not at all be surprising since the optimum DQPSK receiver [10, Chap. 7] makes differential decisions based on an observation of two *symbol* intervals (four bit intervals) whereas the DOQPSK receiver makes differential decisions based on an observation of two *bit* intervals.

B. General Asymptotic Behavior

Following an analogous procedure to that in [1], it can be shown that the maximum value of δ of (4.4) is given by $|\delta|_{\max} = \sqrt{(N-1)^2+1}$ and thus the average BEP is approximately upper bounded by

$$P_b(E) \lesssim \left(\sum\sum_{\mathbf{c}\neq\hat{\mathbf{c}}} w(\mathbf{c},\hat{\mathbf{c}})P(\mathbf{c},\hat{\mathbf{c}})\right)\frac{1}{N-1}\frac{1}{2^{N-1}}\sqrt{\frac{N+\sqrt{(N-1)^2+1}}{8\sqrt{(N-1)^2+1}}} \times \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}\left(N-\sqrt{(N-1)^2+1}\right)}\right) \tag{4.8}$$

where the $w(\mathbf{c},\hat{\mathbf{c}})P(\mathbf{c},\hat{\mathbf{c}})$ terms in the double summation correspond only to those error sequence pairs that result in $|\delta|_{\max}$.

Fig. 2 is an illustration of the asymptotic upper bound on average BEP as computed from (4.8) versus E_b / N_0 in dB and parameterized by the sequence length N. As was the case in [1], the largest improvement in performance is obtained for the first few increases in the value of N with diminishing returns from then on. Since the asymptotic coding gain is obtained from the argument of the complementary error function, we see that, for arbitrary N, this gain (in dB) is given by

$$G = 10\log_{10} \frac{N - \sqrt{(N-1)^2 + 1}}{2 - \sqrt{2}} \tag{4.9}$$

Thus, for $N = 4$, the coding gain is 1.554 dB which therefore represents an asymptotic SNR loss of only 1.446 dB relative to the optimum DQPSK receiver *based on the same observation interval.*[2] In the limit of large N, the coding gain of (4.9) becomes

$$\lim_{N\to\infty} G = 10\log_{10} \frac{1}{2-\sqrt{2}} = 2.323 \tag{4.10}$$

which is now only 0.677 dB away from optimum two-symbol observation DQPSK performance. Of course, one can always apply multiple symbol differential detection to QPSK to also improve its performance as discussed in [1] which in the limit of large observation time approaches the average BEP performance of *coherent detection* of differentially encoded BPSK (or QPSK). Also, since the asymptotic performance of conventional (two-symbol observation) optimum DQPSK is also 2.323 dB worse than coherent detection of differentially encoded BPSK (or QPSK), we conclude that the limiting asymptotic behavior of DOQPSK as considered in this paper is at most 3 dB worse than the latter.

V. Conclusions

Based on a CPM representation for differentially encoded offset QPSK, we have derived and given the average BEP performance of a receiver that performs differential detection of this modulation. Since the receiver is derived from ML considerations, it is expected to be the most power efficient of its type. Based on its resemblance to multiple symbol detection of nonoffset QPSK, the performance of the receiver continues to improve as a function of the observation length (as measured in bit intervals) of the received signal. When compared to the optimum DQPSK receiver which bases its decision on the difference of two symbols, thus requiring observation of the received signal over two symbol (or equivalently, four bit) intervals, the proposed DOQPSK receiver with a 4-bit observation has an asymptotic SNR penalty of 1.446 dB. In the limit of large SNR, whereas multiple symbol differential detection of QPSK approaches the performance of coherently detected BPSK with differential encoding, multiple bit differential detection of OQPSK has a similar limiting behavior but with a penalty of 3 dB. The same limiting behavior has also been demonstrated for spectrally shaped OQPSK [11,12] with linear phase variation. The development of the theory for this modulation scheme is omitted here because of space limitation but is reported in [13] which also includes a comparison with previous ad hoc methods [2].

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[2] Recall that the optimum DQPSK receiver [10, Fig. 7.1] makes its decisions by examining the difference of two symbol decisions and thus its observation interval is $2T_s = 4T_b$.